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transits formerly employed was 58, but of these it has appeared desirable to reject 4; and, from the remaining 54, the latitude deduced by means of the new formulæ is 53° 24′ 47″·40, with a probable error of 0″·10. This result is less by 0″·32 than that given in vol. xiii. of the *Monthly Notices.*

The number of stars employed is sixteen; and, for each of these, Mr. Hartnup has given, in a tabular shape, the values of the constants required in the use of the formulæ; and, in a second table, he has given the individual result derived from every transit of each star, together with the values of the errors of deviation and collimation for each evening, and the corrections which they entail on the results.

This communication, as well as the preceding, will appear in full in the *Memoirs.*

On a Proposed Mode of relieving the Weight of the Moving Portion of an Altazimuth, and on the Theory of the Stability and Pressure exercised by the Upper Works. By Lieut. J. F. Tennant, Bengal Eng., F.R.A.S, First Asst. Gt. Trig. Survey of India.

"The form of altazimuth instruments of which I purpose writing, is that in which the whole of the azimuthal axis is below the vertical circle. These are generally supposed to be less stable and less adapted for very accurate observation than those in which the vertical axis is divided and held both at top and bottom, the telescope and vertical circle being between the two supports. My attention has been especially directed to the class of instruments I have named, as that form is alone suited to theodolites for Geodetical operations, and has led to a similar construction of altazimuths for the trigonometrical survey of India.

"Instruments of this form may be subdivided into two classes, of which the first has the axes *turned down* (the acute end down), and the second* has them *turned* up. The former construction allows of a long axis, while the height of the instrument is not increased, and has been employed for this reason, as length of axis appears to ensure steadiness; but I shall, I think, show reason for preferring the second construction, or that with a moderately long turned up axis.

"I purpose first considering the distribution of weight on an axis supposed to be composed of two conical portions, whose vertical angles are different, and shall first investigate the amount of weight which will be supported on a right cone with the axis vertical by a given pressure on each unit of its surface.

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* An illustration of this construction will be found in Pearson's *Int. to Practical Ast.*, Plate xix.—Editor.

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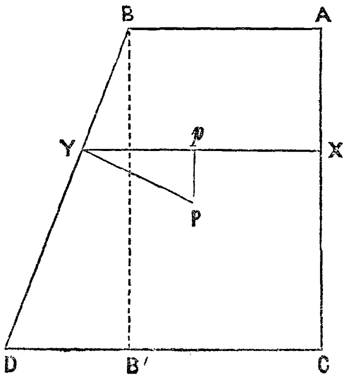
"Let the accompanying figure represent the trapezoid, by whose revolution about A C the frustrum of a right cone forming the axis is generated. Let



$$AB = a = CB', \qquad CD = b, \qquad AC = l,$$

and the angle $DBB' = d$.

"Also let $AX = x$, $XY = y$.

"Also let YP represent the pressure sustained by the element of the surface, whose sides are $d\,(BY)$ and $y\,d\,\theta$ (to use the infinitesimal notation).

$$\text{Now } BY : AX :: BD : AC :: 1 : \cos\alpha,$$

$$\therefore BY = x \sec\alpha,$$

$$\text{and} \quad d\,(BY) = \sec\alpha\, d\,x.$$

"And if p be the pressure per square unit of surface, then $PY = p\,.\,y \sec\alpha\, d\,\theta\, d\,x$, and the resolved part in a vertical direction, or the weight it will support, is

$$Pp = PY \sin\alpha = p\,.\,y \tan\alpha\, d\,\theta\, d\,x,$$

but if we suppose the whole frustrum to be in action, the supported weight will be

$$\int_0^l \int_0^{2\pi} p\,.\,y \tan\alpha\, d\theta\, dx$$

or since $y = x \tan\alpha + a$

$$= \int_0^l \int_0^{2\pi} p \left\{ x \tan^2\alpha + a \tan\alpha \right\} d\theta\,.\,dx,$$

$$= \quad p \int_0^{2\pi} \left(\frac{1}{2}\, l^2 \tan^2\alpha + a\, l \tan\alpha \right) d\,\theta$$

$$= 2\,p\,\pi \left(\frac{l^2}{2} \tan\alpha + a\, l \right) \tan\alpha$$

$$\text{But } l \tan\alpha + a = b, \text{ and } l \tan\alpha = b - a$$

$$\therefore \text{weight} = p\,\pi\,(b + a)\,(b - a) = \pi\,(b^2 - a^2)\,p$$

whence if W be the total weight supported by the frustrum, we shall have

$$p = \frac{W}{\pi\,(b^2 - a^2)}$$

or it will have the same value as it would have were a similar weight supported on the base of the inclined surface in action.

"I have supposed the pressure to be uniformly distributed over the acting surface, which would, I conceive, be the case had we a true mathematical frustrum; and I think this will, in a

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well-made axis, be an approximate representation of the case. We are not, however, entitled to consider that, if we have two frustra (the angle β in the second being substituted for α) simultaneously in action, that the pressure on the surfaces will be equal in both.

We can, of course, adjust this to some extent as we may please originally, but the adjustment will only be permanent, if the pressure on the frustrum, the angle of whose cone is 2α (which I shall call the frustrum 2α), be to the pressure on the frustrum 2β in the proportion requisite to ensure wear keeping the same state of fitting. Now, the amount of wear on the frustrum 2α will be to the amount of wear on the frustrum 2β as $\sin\alpha$ to $\sin\beta$, when this state is attained; and if we assume that the wear is proportional to the pressure, an assumption which, in default of positive data, seems to be probable, we shall have

$$P_\alpha \text{ (or pressure on frustrum } 2\alpha) : P_\beta :: \sin\alpha : \sin\beta$$

" If there be a flange, or $\beta = 90^\circ$,

$$P_\alpha : P \text{ on flange} :: \sin\alpha : 1$$

" Let us, on this supposition, assume $p \sin\beta$ as the surplus pressure on the more obtuse cone, then will $p \sin\alpha$ be that on the more acute one; and we shall have, calling A and B the bases of the acting surfaces,

$$p\,A \sin\alpha + p\,B \sin\beta = W, \text{ the free weight,}$$

but P the total pressure causing friction

$$= p \sin\alpha \,.\, A \operatorname{cosec}\alpha + p \sin\beta.\ B \operatorname{cosec}\beta \text{ or } p\,(A + B)$$

whence,

$$P = W \frac{A + B}{A \sin\alpha + B \sin\beta}$$

which, when α is a very small angle, as is usual, will very nearly equal W cosec β.

" This explains the theory of the flange and Ramsden's cup in rendering motion free, which would otherwise be so stiff as to be objectionable.

" Keeping in view these principles, we may, from a knowledge of the co-efficient of friction derived from experiment, predict the force which would be sufficient, at the place where the hand is intended to be applied, in moving the instrument to cause it to revolve.

" Where this force is too large, it becomes necessary to relieve a portion of the weight of the moving part. So far as I am aware, this has hitherto been accomplished by springs, and by counterpoising weights acting on levers; friction rollers in each case being the mode of transmitting the action to the upper works. Variable tension at different times, and consequently irregularly applied relief, is an objection to springs; and, moreover, from the

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necessary shortness, they generally have to be very stiff, and small differences of compression are correspondent to material pressures. These are avoided by counterpoises, but they add to the weight of the instrument; and where (as in theodolites) the instrument travels, they must be previously removed, of which perhaps the inconvenience can only be appreciated by those who have had to dismantle and pack up instruments late at night, when wearied with a long day's work. In either case, want of power to render the resultant transmitted pressure *axial* is a great objection, and it does not appear that this can be accurately managed.

" It seems absolutely essential to the good performance of an instrument of the class I am examining, that the resultant of the weight of the moving portions and relieving springs should be *axial*, as otherwise there will not be fair action on the axis; and the simplest way of attaining this would appear to be, to make the weight and relieving force each axial separately. I also think that a low position of the centre of gravity, such as will enable it to be *immediately supported* by the axis, and thus secure the instrument against any great tendency to oscillatory motion, is at all events very advisable.

" The weight of a telescope and vertical circle, and the height to which they must be raised to allow of reversion of the telescope, is no doubt an obstacle to carrying out the last principle; but we can still lower the centre of gravity within the axis, by turning up the axis and using a very heavy drum, to which the only objections I know are the want of portability and the difficulty of sufficiently reducing friction. The former is of less importance than the steadiness of the instrument, and the latter, I believe, can generally be sufficiently relieved by the means I am about to mention.

" I propose, then, to adapt to the upper part of the hollow conical axis a shallow reservoir for oil, having a narrow cylindrical tube rising out of it, in which works a piston urged by a weight. The reservoir is to be open downwards, by which pressure will be transmitted by the oil to the head of the solid cone, and the same pressure will assist in lubrication. We may then make this pressure relieve any amount of weight within the limits I shall point out, and at the same time secure its rigorously axial application.

" If the free weight be m W, p the pressure on the unit of the flange, and $p \sin \alpha$, therefore, that on the conical frustrum before the relief is applied, or while the weight is W, then $mp \sin \alpha$ will be the pressure on the relief being given, unless the pressure on a unit of surface on the head of the axis be greater than $mp \sin \alpha$, when the whole surface of the frustrum will come into play, and we must consider the relieving pressure as acting on the area of its base.

" It is quite evident, however, that if q be the relieving pressure on a unit of surface, mp must be $> q$ as a necessary condition

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to prevent the expulsion of the oil when the capillary action which retains it is overcome. In cases where there is no flange, the pressure on the unit of surface of the cone will have to exceed that on the head for the same reason, and if the necessary relief cannot be given with the given dimensions of the axis, it can of course in this case be attained by increasing its dimensions.

"How far attention to these principles will secure stability in conjunction with freedom of motion, it is impossible to say without putting them in practice, but I hope soon to have an opportunity of carrying them out in endeavouring to obtain a better azimuthal motion for one of the 3-foot vertical circles belonging to the trigonometrical survey of India, which, though producing results of the highest order, would still admit of an improvement. I am induced to offer these remarks to the Society, in the hope that the hydraulic relief may be an improvement on those previously in use, and that by it a class of instruments in which I am professionally much interested may be rendered more perfect.

"I have not attempted to lay down any rules as to the dimensions of axes or their forms. So many variables are involved in the determination of the best form, and a departure from it to a moderate extent will so little affect the value of instruments, that probably no definite rules could be so valuable as the intuitive perception of the practised artist. I believe, however, the general principles I have here laid down, and the theory to guide their application, will occasionally be found useful, and they may tend to place the form of instrument preferred by Mr. Troughton on equal terms with that chosen by some other makers, by increasing the steadiness without the disadvantage of unequal expansions inseparable probably from the framework required to support an axis at both extremities.

"In the case of the 3-foot circle I propose that the drum pillars and micrometer-supporters should be cast in one piece of iron, and, after being fitted, adjusted so as to have the centre of gravity axial by floating the mass in mercury, and adding or subtracting small weights of metal, as may appear desirable.

"*Masuri, June* 4, 1856."

Notes on the Manufacture of Tubes for Refracting Telescopes.

By Frederick Brodie, Esq.

"In the early part of this year I parted with my Munich refractor of 6$\frac{1}{3}$ inches clear aperture, and obtained from the Rev. W. R. Dawes his American object-glass, made by Mr. Alvan Clarke, of Boston, U. S., an account of which was published in the *Monthly Notices*, vol. xv. page 79.

"The original tube for this glass was retained by Mr. Dawes for mounting another larger object-glass, by the same maker. It,